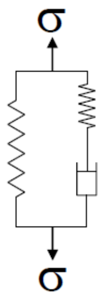

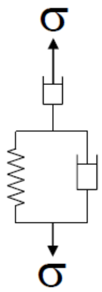

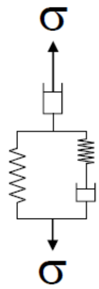

Standard linear solid

Time domain:

$$\sigma(t) = k_e \varepsilon(t) + k_1 \varepsilon_{e1}(t)$$

$$k_1 \varepsilon_{e1}(t) = \eta(\dot{\varepsilon}(t) - \dot{\varepsilon}_{e1}(t))$$

$$\sigma(t) = \sigma_0 u(t)$$

Laplace transform:

$$\bar{\sigma}(s) = k_e \bar{\varepsilon}(s) + k_1 \bar{\varepsilon}_{e1}(s)$$

$$k_1 \bar{\varepsilon}_{e1}(s) = \eta s(\bar{\varepsilon}(s) - \bar{\varepsilon}_{e1}(s))$$

$$\bar{\sigma}(s) = \frac{\sigma_0}{s}$$

$$(k_1 + \eta s)\bar{\varepsilon}_{e1}(s) = \eta s \bar{\varepsilon}(s)$$

$$\bar{\varepsilon}_{e1}(s) = \frac{\eta s}{k_1 + \eta s} \bar{\varepsilon}(s)$$

$$\bar{\varepsilon}_{e1}(s) = \frac{s}{\frac{k_1}{\eta} + s} \bar{\varepsilon}(s)$$

k_1 is a stiffness : Pa

η is a viscosity : Pa.s

$\frac{k_1}{\eta}$ is the inverse of a time : s^{-1}

Characteristic time: $\tau = \frac{\eta}{k_1}$

$$\bar{\varepsilon}_{e1}(s) = \frac{s}{\frac{1}{\tau} + s} \bar{\varepsilon}(s)$$

$$\bar{\sigma}(s) = \left[k_e + \frac{k_1 s}{\frac{1}{\tau} + s}\right] \bar{\varepsilon}(s)$$

$$\bar{\varepsilon}(s) = \sigma_0 \frac{1}{k_e s + \frac{k_1 s^2}{\frac{1}{\tau}+s}}$$

$$\bar{\varepsilon}(s) = \sigma_0 \frac{\frac{1}{\tau}+s}{\frac{k_e}{\tau}s + k_e s^2 + k_1 s^2}$$

$$\bar{\varepsilon}(s) = \sigma_0 \frac{\frac{1}{\tau}}{s\left[\frac{k_e}{\tau}+(k_e+k_1)s\right]} + \frac{s}{s\left[\frac{k_e}{\tau}+(k_e+k_1)s\right]}$$

$$\bar{\varepsilon}(s) = \sigma_0 \frac{\frac{1}{\tau(k_e+k_1)}}{s\left[\frac{k_e}{\tau(k_e+k_1)}+s\right]} + \frac{\frac{1}{(k_e+k_1)}}{\left[\frac{k_e}{\tau(k_e+k_1)}+s\right]}$$

$$a = \frac{k_e}{\tau(k_e+k_1)}$$

Time domain (again!):

$$\varepsilon(t) = \frac{\sigma_0}{k_e}\left[1 - e^{-\frac{k_e t}{\tau(k_e+k_1)}}\right] + \frac{\sigma_0}{k_e+k_1} e^{-\frac{k_e t}{\tau(k_e+k_1)}}$$

$$\varepsilon(t) = \frac{\sigma_0}{k_e} + \left[\frac{\sigma_0}{k_e+k_1} - \frac{\sigma_0}{k_e}\right] e^{-\frac{k_e t}{\tau(k_e+k_1)}}$$

$$\varepsilon(t) = \left[\frac{1}{k_e} - \frac{k_1}{k_e(k_e+k_1)} e^{-\frac{k_e t}{\tau(k_e+k_1)}}\right]\sigma_0$$

At time 0

$$\varepsilon(0) = \frac{\sigma_0}{k_e+k_1}$$

$$\dot{\varepsilon}(0) = \frac{k_1 \sigma_0}{\tau(k_e+k_1)^2}$$

Liquid

Time domain:

$$\sigma(t) = \eta\dot{\varepsilon}_1(t) = k_e[\varepsilon(t) - \varepsilon_1(t)] + \eta[\dot{\varepsilon}(t) - \dot{\varepsilon}_1(t)]$$

$$\sigma(t) = \sigma_0 u(t)$$

Laplace transform:

$\bar{\sigma}(s) = \eta s \bar{\varepsilon}_1(s) = k_e[\bar{\varepsilon}(s) - \bar{\varepsilon}_1(s)] + \eta s[\bar{\varepsilon}(s) - \bar{\varepsilon}_1(s)]$

$\bar{\sigma}(s) = \eta s \bar{\varepsilon}_1(s) = (k_e + \eta s)[\bar{\varepsilon}(s) - \bar{\varepsilon}_1(s)]$

$\bar{\sigma}(s) = \frac{\sigma_0}{s}$

$(2\eta s + k_e)\bar{\varepsilon}_1(s) = (k_e + \eta s)\bar{\varepsilon}(s)$

$\bar{\varepsilon}_1(s) = \frac{\eta s + k_e}{2\eta s + k_e}\bar{\varepsilon}(s)$

$\bar{\varepsilon}_1(s) = \frac{s + 1/\tau}{2s + 1/\tau}\bar{\varepsilon}(s)$

$\frac{\sigma_0}{s} = \eta s \frac{s + 1/\tau}{2s + 1/\tau}\bar{\varepsilon}(s)$

$\sigma_0 = k_e \tau\, s^2 \frac{s + 1/\tau}{2s + 1/\tau}\bar{\varepsilon}(s)$

$\bar{\varepsilon}(s) \quad = \frac{\sigma_0}{k_e \tau}\frac{2s + 1/\tau}{s^2(s + 1/\tau)}$

$\bar{\varepsilon}(s) \quad = \frac{\sigma_0}{k_e \tau}\left[\frac{2}{s(s + 1/\tau)} + \frac{1/\tau}{s^2(s + 1/\tau)}\right]$

Time domain (again!):

$\varepsilon(t) = 2\frac{\sigma_0}{k_e}\left[1 - e^{-\frac{t}{\tau}}\right] + \frac{\sigma_0}{k_e \tau^2}\left[t\tau - \tau^2\left(1 - e^{-\frac{t}{\tau}}\right)\right]$

$\varepsilon(t) = \frac{\sigma_0}{k_e}\left[1 - e^{-\frac{t}{\tau}}\right] + \frac{\sigma_0 t}{k_e \tau}$

$\varepsilon(0) = 0$