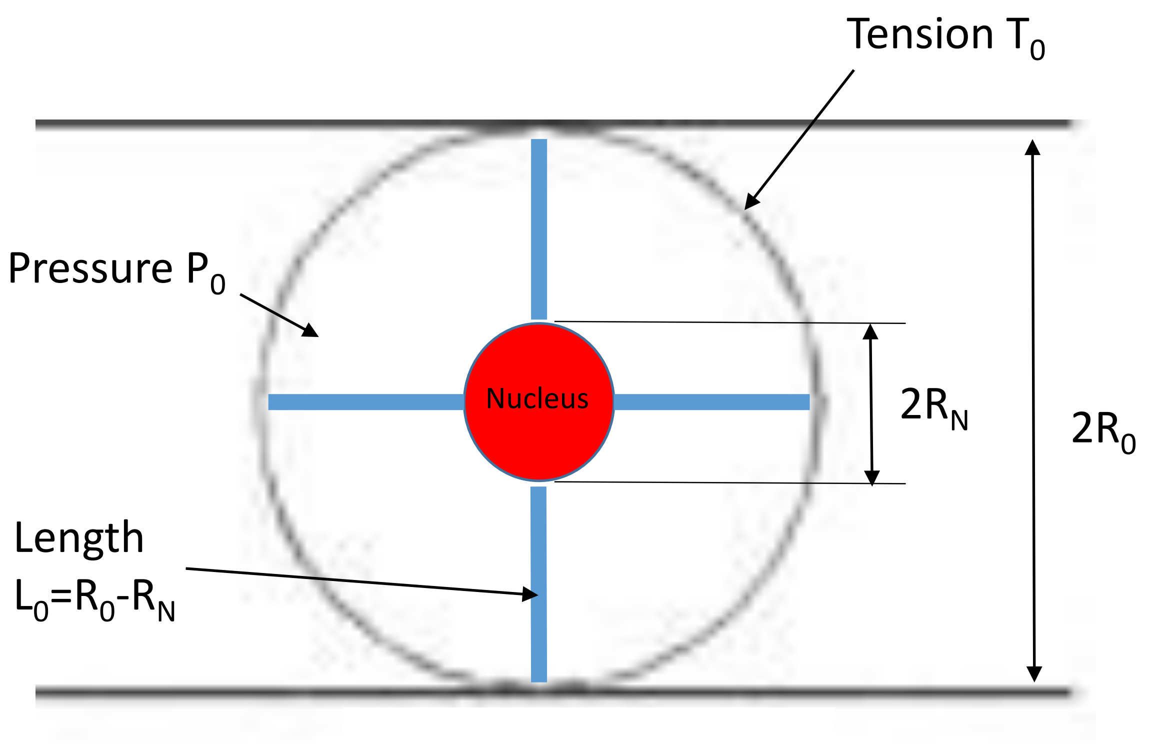
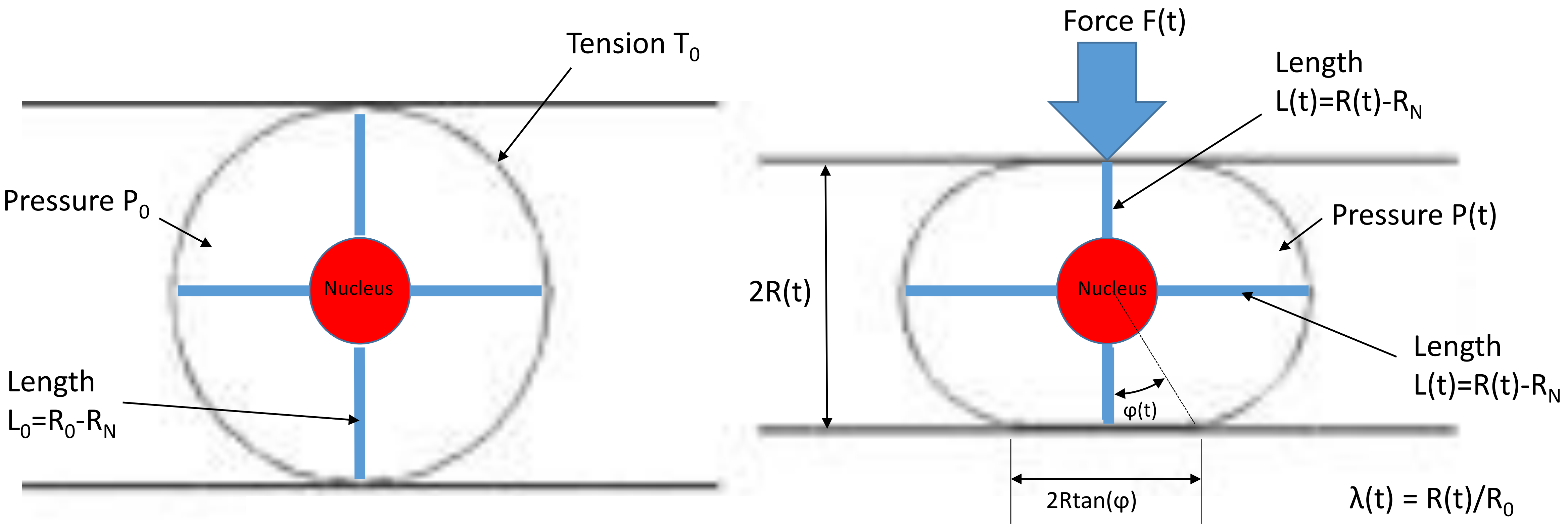

Tension T_0
Pressure P_0
Nucleus
$2R_N$
$2R_0$
Length
$L_0=R_0-R_N$



Tension T0
Pressure P0
Nucleus
Length
L0=R0-RN
Force F(t)
Length
L(t)=R(t)-RN
Pressure P(t)
2R(t)
Nucleus
Length
L(t)=R(t)-RN
φ(t)
2Rtan(φ)
λ(t) = R(t)/R0

Force

stresses

strains

Displacement
2 unknowns
R(t)
φ(t)

3 components:

Vertical Fibers
Horizontal fibers
Bulk : P(t)
Membrane

Equilibrium at the contact:

$P \pi R^2 \tan^2(\varphi) + \sigma_{fv} A_f = F$

Equilibrium at the apex:

$T + h_0 \sigma_{fh} = PR/2$

Inject the expressions relating the stresses to R(t) and φ(t) in these equations

Then I have 2 non linear equations of 2 unknowns

Numerical resolution

Force → stresses → strains → Displacement

4 components:

Vertical Fibers
Horizontal fibers
Bulk
Membrane

membrane

$$\Psi = \int T \, dE$$

$$T = \partial \Psi / \partial E = Y (E+E')$$

Force → stresses → strains → Displacement

4 components:

Vertical Fibers
Horizontal fibers
Bulk
Membrane

fibers

$\Psi = \int \sigma_f \, dE$

$\sigma_f = \partial \Psi / \partial E$

Force → stresses → strains → Displacement

4 components:

Vertical Fibers
Horizontal fibers
Bulk
Membrane

bulk

$\Psi = \int P \, dJ$

$P = \partial \Psi / \partial J = K(J-1)$

Force → stresses → strains → Displacement
2 unknowns
$R(t)$
$\varphi(t)$

4 components:

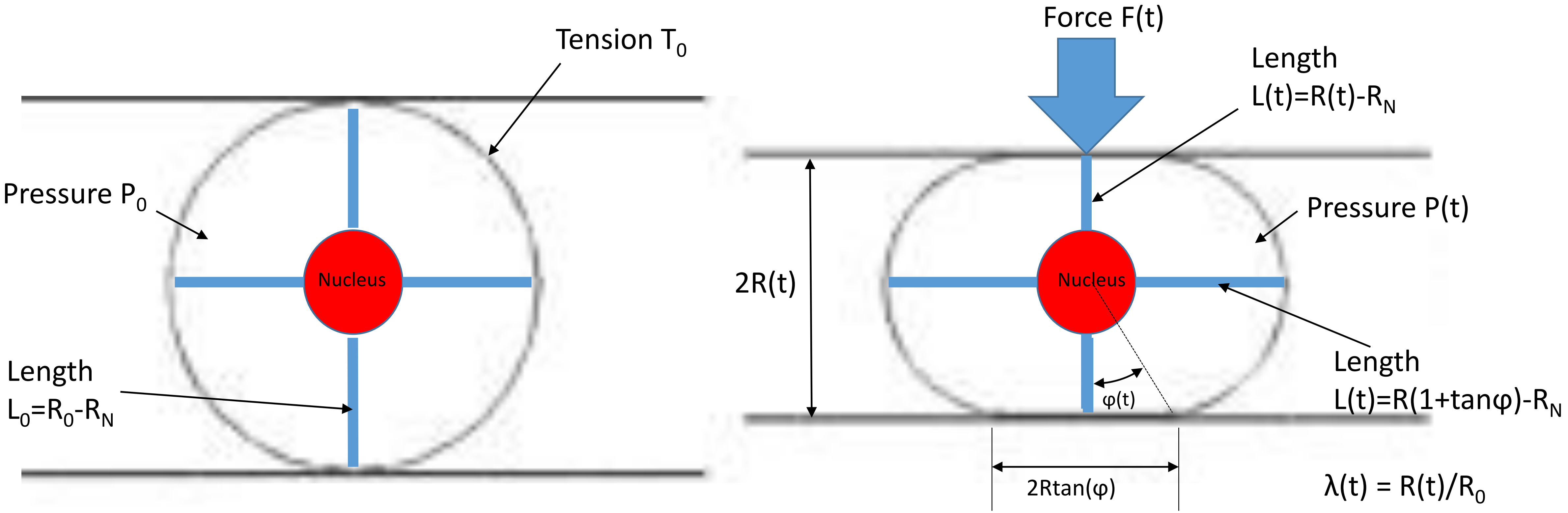
Vertical Fibers
Horizontal fibers
Bulk
Membrane

$$J = V/V_0 = (R/R_0)^3 \, [3/2 \tan^2(\varphi) + 3\pi/4 \tan(\varphi) + 1 \,]$$

$$E = (R/R_0)^2 \, [\tan(\varphi) + \tan^2(\varphi)/2]$$

$$E_{fv} = [\, [(R-R_N)/(R_0-R_N)]^\wedge 2 - 1] \,/2$$

$$E_{fh} = [\, [(R(1+\tan\varphi)-R_N)/(R_0-R_N)]^\wedge 2 - 1] \,/2$$



For the membrane

Energy per unit area

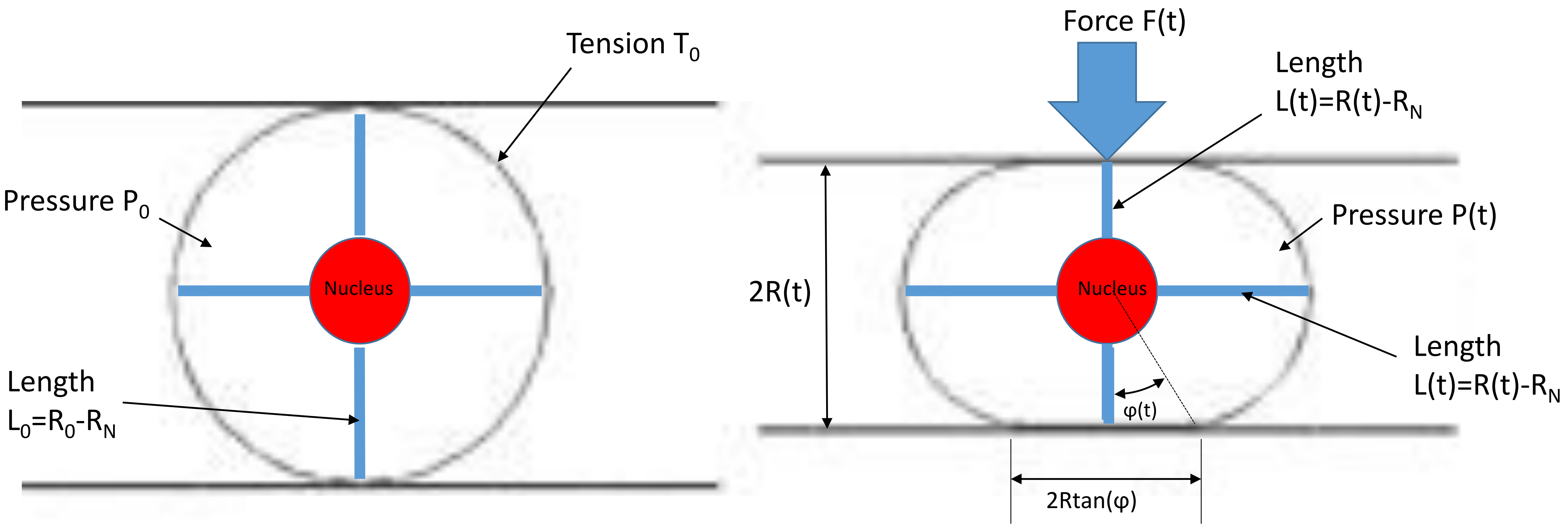
$\Psi(E)=Y/2\ (E+E')\hat{}2$

E' is the prestrain

$E = S/S_0 - 1$

$S = 4\pi R^2\ [1+\tan(\varphi)+\tan^2(\varphi)/2]$

$S_0 = 4\pi R_0^2$



For the bulk

$$\Psi(J)=K/2\ (J-1)^2$$

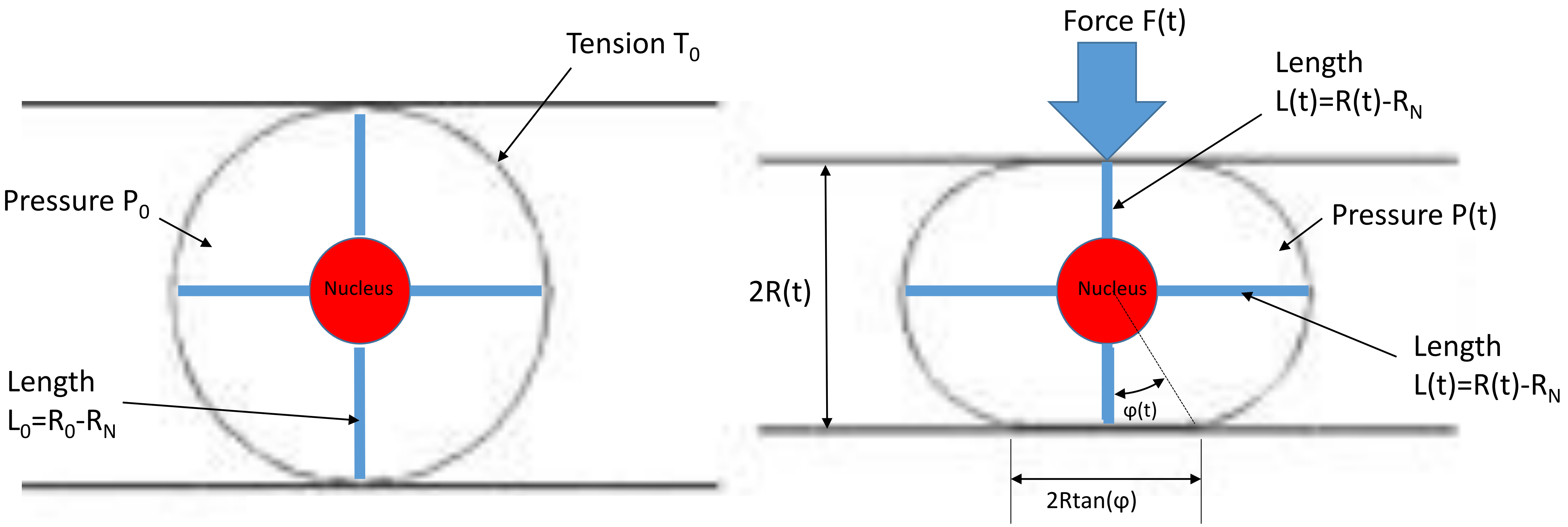
$$J=V/V_0$$

$$V = V_0$$

$$V = 2\pi R^3 \tan^2(\varphi)+ \pi^2 R^3 \tan(\varphi)+ 4/3\ \pi R^3$$

$$V = 4/3\ \pi R^3\ [3/2\tan^2(\varphi) + 3\pi/4 \tan(\varphi)+ 1\ ]$$

$$V_0 = 4/3\ \pi R_0^{\ 3}$$



For the fibers

$\Psi(E)=k/2\ E^2$ if $E<0$
$\Psi(E)=k/2a\ [\ \exp(aE^2) - 1]$ if $E<0$

$E = [\ (L/L_0)^\wedge 2 - 1]\ /2$

$$\mathrm{Wtot}(R,\mathrm{phi}) = \Psi\mathrm{fh}(R,\mathrm{phi})\mathrm{Vfh} + \Psi\mathrm{fv}(R,\mathrm{phi})\mathrm{Vfv} + \Psi\mathrm{m}(R,\mathrm{phi})\mathrm{Vm} + \Psi\mathrm{b}(R,\mathrm{phi})\mathrm{Vb} + \Psi\mathrm{n}(R,\mathrm{phi})\mathrm{Vn} + \mathrm{int}(FdR)$$

$$\mathrm{dWtot} = \mathrm{DWtot}/\mathrm{DR}\ dR + \mathrm{DWtot}/\mathrm{Dphi}\ \mathrm{dphi} = 0$$

$$\mathrm{DWtot}/\mathrm{DR} = 0$$

$$D\Psi\mathrm{fh}/\mathrm{DE11} * \mathrm{Vfh} * \mathrm{DE}/\mathrm{DR} + \mathrm{DWfv}/\mathrm{DR} + \mathrm{DWm}/\mathrm{DR} + \mathrm{DWb}/\mathrm{DR} + \mathrm{DWn}/\mathrm{DR} + F = 0$$

$$S11 * \mathrm{Vfh} * \mathrm{DE}/\mathrm{DR} + \ldots$$

$$\mathrm{DWtot}/\mathrm{Dphi} = 0$$

$$\mathrm{DWfh}/\mathrm{Dphi} + \mathrm{DWfv}/\mathrm{Dphi} + \mathrm{DWm}/\mathrm{Dphi} + \mathrm{DWb}/\mathrm{Dphi} + \mathrm{DWn}/\mathrm{Dphi} = 0$$